UG/1-Sem/H/19

2019

MATHEMATICS

(Honours)

Paper : MTMHI-DC-01

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.

Group - A

1. Answer any *four* questions : 1×4=4

(a) Show that $\lim_{x \to 0} \sin\frac{1}{x}$ does not exist.

(b) Find the range of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards.

(c) Prove that $\int_0^\infty \sqrt{x}\, e^{-x^3}\, dx = \frac{\sqrt{\pi}}{3}$.

P.T.O.

(d) Find the radius and centre of the circle $r = 3\sin\theta + 4\cos\theta$.

(e) Find the eccentricity and the vertex of the conic $r = \frac{6}{1-\cos\theta}$.

(f) When the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of straight line?

(g) Determine the equation of the sphere described on the join of $(2,-3,4)$ and $(-1,0,5)$ as diameter.

Group - B

Answer any *two* questions. $5\times2=10$

2. If $\log y = \tan^{-1}x$, then prove that

$$(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0$$

3. If $I_n = \int_0^{\pi/2} x^n \sin x\, dx$, $n > 1$, show that

$$I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$

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Hence evaluate $\int_0^{\pi/2} x^5 \sin x \, dx$.

4. Reduce the equation

$3x^2 + 2xy + 3y^2 - 4\sqrt{2}x - 4\sqrt{2}y = 0$ to its canonical form and determine the nature of the conic.

5. Prove that the centre of spheres, which touches the straight line $y = mx, z = c; y = -mx, z = -c$ lie on the surface $mxy + cz(1+m^2) = 0$.

Group - C

Answer any *two* questions : 9×2=18

6. (a) A function $f : \mathbb{R} \to \mathbb{R}$ satisfies the condition $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, prove that either $f(x) = 0$ or $f(x) > 0$ for all $x \in \mathbb{R}$. 3

(b) Find the value of

$$\lim_{n\to\infty}\left(\frac{1}{2n+1} + \frac{1}{2n+2} + \cdots + \frac{1}{6n}\right).$$ 3

P.T.O.

(c) Obtain the equation to the generators of the paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ through a point of the principal parabolic section. 3

7. (a) Find the asymptotes of the curve

$$x^3-2y^3+2x^2y-xy^2+xy-y^2+1=0.$$ 3

(b) Show that the distance between the parallel lines represented by

$ax^2+2hxy+by^2+2gx+2fy+c=0$ is

$2\sqrt{\frac{g^2-ac}{a(a+b)}}$. 3

(c) Find the co-ordinates of the vertex, focus and length of latus rectum of the principal sections of the hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{2z}{c}$. 3

8. (a) Find the points of inflexion on the curve $(\theta^2-1)r=a\theta^2$. 3

(b) Find the total length of the curve

$\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$. 3

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(c) The normal at a variable point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the diameter CD, conjugate to CP, at Q. Show that the locus of Q is $\frac{a^2}{x^2}+\frac{b^2}{y^2}=\left(\frac{a^2-b^2}{x^2+y^2}\right)^2$, C being the centre of the ellipse. 3

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