

UG/1-Sem/H/19

2019

MATHEMATICS

(Honours)

Paper : MTMHI-DC-02

(Algebra)

[CBCS]

Full Marks : 32

Time : Two Hours

The figures in the margin indicate full marks.

Group - A

1. Answer any *four* questions : $1 \times 4 = 4$

(a) Show that $f(x) = 2x^4 - 7x^3 + 8ax^2 - 3bx + 17$

be divisible by $x+2$ if $32a - 6b = 7$.

(b) Find the cube root of -1 .

(c) If $a+b+c=0$ show that $ab+bc+ca \leq 0$

(d) Give an example of a mathematical relation on a given set that is both reflexive and transitive, but not symmetric.

P.T.O.

(2)

(e) Find all eigen values of $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$.

(f) Solve $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(g) Construct a polynomial equation of degree four, with real coefficients, having roots $\pm(x + 3i)$.

Group - B

Answer any two questions.

$5 \times 2 = 10$

2. If $u + iv = \operatorname{cosec}(x + iy)$ then show that

$$\frac{u^2}{\sin^2 x} - \frac{v^2}{\cos^2 x} = \frac{u^2}{\cosh^2 x} + \frac{v^2}{\sinh^2 x} = (u^2 + v^2)^2$$

3. Prove that the equation $(x+1)^4 = a(x^4 + 1)$ is a reciprocal equation if $a \neq 1$ and solve it when $a = -2$.

4. Verify Cayley-Hamilton's theorem for the matrix

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix} \text{ and hence evaluate } A^{-1}.$$

(3)

5. Show that an equivalence relation on a set S determines a partition on the set S and conversely a partition on S yield an equivalence relation on S .

Group - C

Answer any *two* questions : $9 \times 2 = 18$

6. (a) Show that congruence relation is an equivalence relation. 1

- (b) If α, β, γ be the roots of the equation

$x^3 + px^2 + qx + r = 0$ ($r \neq 0$), find the equation whose roots are

$$\alpha\beta + \frac{1}{\gamma}, \beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}.$$

- (c) If $x = \log \tan \left(\frac{\pi}{4} + \frac{y}{2} \right)$ then prove that

$$y = -i \log \tan \left(\frac{\pi}{4} + \frac{x}{2} i \right) \quad 4$$

7. (a) Reduce the following quadratic form to its normal form and find the rank and signature :

$$2x^2 + 4y^2 + z^2 - 4xy - 2yz + 2zx. \quad 5$$

P.T.O.

(4)

- (b) If x_1, x_2, \dots, x_n be n positive rational numbers such that $S = x_1 + x_2 + \dots + x_n$, show that

$$\left(\frac{S}{x_1} - 1\right)^{x_1} \left(\frac{S}{x_2} - 1\right)^{x_2} \dots \left(\frac{S}{x_n} - 1\right)^{x_n} \leq (S-1)^S. \quad 4$$

8. (a) If $\alpha, \beta, \gamma, \delta$ be the roots of the equation $x^4 + px + q = 0$ then find $\sum \alpha^4$. 2

- (b) For what values of λ the following system of equations are consistent? Solve in each consistent case.

$$\begin{aligned}x - y + z &= 1 \\x + 2y + 4z &= \lambda \\x + 4y + 6z &= \lambda^2.\end{aligned}$$

4

- (c) If a, b are prime to each other and $a|c, b|c$ then prove that $ab|c$. 3
