2020

MATHEMATICS (Honours)

Paper Code : VIII - A & B

(New Syllabus)

Full Marks : 60

Time : Three Hours

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code : | III | A | & | B |

  Subject Name : [ ]

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code : | III | A | & | B |

Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :
1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Turn Over

Paper Code : VIII - A

Full Marks : 10 Time : Fifteen Minutes

Choose the correct answer.

Each question carries 2 marks.

1. The inverse Laplace transform of the function $\frac{1}{(s+a)^3}$, $s > -a$, a being a scalar, is

   (A) $t^2 e^{at}$

   (B) $\frac{1}{2} t^2 e^{at}$

   (C) $\frac{1}{2} t^2 e^{-at}$

   (D) $\frac{1}{2} t^3 e^{-at}$.

2. For any three elements a, b and c in a Boolean algebra $\mathcal{B}$, which of the following holds?

   (A) $a \cdot b + a \cdot (a \cdot b)' = b$

   (B) $a \cdot b + a \cdot (a \cdot b)' = a'$

   (C) $a \cdot b + a \cdot (a \cdot b)' = 1$

   (D) $a \cdot b + a \cdot (a \cdot b)' = a$.

3. If A_i and B_i are two covariant vectors and g^{ij} is a contravariant metric tensor in a Riemannian space V_n, then $g^{ij}(A_i B_j - A_j B_i)$ is equal to

   (A) 1

   (B) -1

   (C) 0

   (D) 2.

Turn Over

4. If T is a linear operator on a two-dimensional real vector space $V_2(\mathbb{R})$ defined by $T(a, b) = (0, a)$ for all $(a, b) \in V_2(\mathbb{R})$, which of the following is true?

(A) $T^2 = 0$ and $T = 0$

(B) $T^2 = 0$ but $T \neq 0$

(C) $T^2 \neq 0$ but $T = 0$

(D) $T^2 \neq 0$ and $T \neq 0$.

5. The total number of normal subgroups of the Klein 4-group K_4 is

(A) 2

(B) 3

(C) 4

(D) 5.

Turn Over

2020

MATHEMATICS (Honours)

Paper Code : VIII - B

(New Syllabus)

Full Marks : 50 Time : Two Hours Forty Five Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

1. Answer any *two* of the following questions: $4 \times 2 = 8$

(a) Define the kernel of a linear transformation. Test whether the transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ defined by $T(a, b, c) = (a + 1, b + 1, c + 1)$ for all $(a, b, c) \in \mathbb{R}^3$ is linear or not. $1 + 3$

(b) Let V be the vector space of all 2×2 matrices over $\mathbb{R}$, and let $M = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$. Let $T : V \longrightarrow V$ be a linear transformation defined by $T(A) = AM - MA$ for all $A \in V$. Find the kernel of T. Also find a basis of the kernel of T. $3 + 1$

(c) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation defined by

$$T(x, y, z) = (2x + y - z, y + 4z, x - y + 3z)$$

for all $(x, y, z) \in \mathbb{R}^3$. Find the matrix of T relative to the ordered basis

$$\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$$

of $\mathbb{R}^3$. 4

2. Answer any *two* of the following questions: $3 \times 2 = 6$

(a) Let G be a group and H be a subgroup of G. If for all $a, b \in G$, $ab \in H \implies ba \in H$, prove that H is a normal subgroup of G.

(b) Prove that two finite cyclic groups of the same order are isomorphic.

(c) Let G and G' be two groups and let $f : G \longrightarrow G'$ be an epimorphism. Prove that if G is commutative, then G' is also commutative.

3. Answer any *two* of the following questions : 3×2=6

(a) Express the Boolean function $(x + y + z)(xy + x'z)'$ in DNF.

(b) Simplify the Boolean expression $(x + y + z')(x + y' + z)$ and draw the corresponding circuit.

(c) In a Boolean algebra $\mathcal{B}$, show that $ab + a'b' + bc = ab + a'b' + a'c$ for all $a, b, c \in \mathcal{B}$.

Turn Over

4. Answer any *three* of the following questions: $5 \times 3 = 15$

(a) Using convolution theorem find $L^{-1}\left\{\dfrac{1}{(s-1)(s-2)}\right\}$.

(b) If $L\{f(t)\} = F(s)$, show that $L\{tf(t)\} = -F'(s)$.

(c) Using Laplace transform solve $\dfrac{d^2y}{dt^2} + y = t$, given $y = 1$, $\dfrac{dy}{dt} = -2$ when $t = 0$.

(d) Find the series solution of $y'' + xy' + x^2y = 0$ near $x = 0$.

(e) Using Laplace transform prove that

$$\int_0^\infty \frac{e^{-t}\sin\sqrt{3}t}{t}\,dt = \frac{\pi}{3}.$$

5. Answer any *three* of the following questions: $5 \times 3 = 15$

(a) Show that the contraction of the outer product of the tensors A^i and B_j is an invariant.

(b) Prove that $\left\{ {i \atop i\,j} \right\} = \dfrac{\partial}{\partial x^j}\left(\log\sqrt{g}\right)$, where $g = |g_{ij}|$.

(c) Show that in a Riemannian space V_4 with the line element

$$ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2,$$

the vector $\left(\sqrt{2}, 0, 0, \frac{\sqrt{3}}{c}\right)$ is a unit vector.

(d) A quantity $X(i,j,k)$ is such that in coordinate system x^i, $X(i,j,k)B_k^{jl} = C_i^l$, where B_k^{jl} is an arbitrary tensor and C_i^l is a tensor. Prove that $X(i,j,k)$ is a tensor of type X_{ij}^k.

(e) Prove that the covariant derivative of a contravariant vector is a tensor of type